Analysis and Efficient Computation for the Schrödinger Equations in the Semiclassical Regimes

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In this talk, I will review recent results on analysis and efficient computation for the linear and nonlinear Schrödinger (NLS) equations in the semiclassical regimes

$$i\,\varepsilon\partial_t\psi^\varepsilon = -\frac{\varepsilon^2}{2}\Delta\psi^\varepsilon + V(\mathbf{x})\psi^\varepsilon + f(|\psi^\varepsilon|^2)\psi^\varepsilon, \tag{1}$$

$$\psi^\varepsilon(\mathbf{x}, t=0) = \psi_0^\varepsilon(\mathbf{x}). \tag{2}$$

Here $\psi^\varepsilon = \psi^\varepsilon(\mathbf{x}, t)$ is the complex-valued wave function, $V = V(\mathbf{x})$ is a given real-valued external potential, f is a real-valued smooth function satisfying $f(0) = 0$, and $0 < \varepsilon \ll 1$ is the scaled (small) Planck constant. We begin with a brief review on the formal semiclassical limit of the NLS by using different approaches including WKB method, Winger transform, Grenier's generalized WKB analysis, etc. A time-splitting spectral (TSSP) method was introduced to efficiently compute the dynamics of the NLS in the semiclassical regimes. The numerical method is explicit, unconditionally stable, time reversible and time transverse invariant. Moreover, it conserves the position density in the discretized level and has the best spatial/temporal resolution for the NLS in the semiclassical regimes. Comparison between the solutions of the NLS and its quantum hydrodynamical limit are presented, especially when the quantum hydrodynamical equations have shocks and/or vacuum. Finally, the analysis and computation results are extended for the NLS with an angular momentum rotation term and coupled nonlinear Schrödinger equations.

References

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