Shock Reflection-Diffraction and Multidimensional Conservation Laws

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Shock reflection-diffraction problems arise not only in many important physical situations but also are fundamental in the mathematical theory of multidimensional hyperbolic systems of conservation laws since their solutions are building blocks and asymptotic attractors of general solutions to the Euler equations for multidimensional compressible fluids. In particular, when a plane shock hits a wedge head on, it experiences a reflection-diffraction process and then a self-similar reflected shock moves outward as the original shock moves forward in time. The complexity of reflection-diffraction configurations was first reported by Ernst Mach in 1878, and experimental, computational, and asymptotic analysis has shown that various patterns of shock reflection-diffraction may occur, including regular and Mach configurations. However, most of the fundamental issues for shock reflection-diffraction have not been understood, including the global structure, stability, and transition of the different patterns of shock reflection-diffraction. Therefore, it is essential to establish the global existence, structural stability, regularity of solutions of shock reflection-diffraction in order to understand fully the phenomena of shock reflection. On the other hand, there has been few rigorous mathematical result on the global issues of shock reflection-diffraction, including the case of potential flow which is widely used in aerodynamics. One of the main reasons is that the problems involve several challenging difficulties in the analysis of nonlinear partial differential equations such as mixed equations of hyperbolic-elliptic type, free boundary problems, and corner singularity where an elliptic degenerate curve meets a free boundary.

In this talk we will start with various shock reflection-diffraction phenomena, their fundamental scientific issues, and their theoretical roles in the mathematical theory of multidimensional hyperbolic systems of conservation laws. Then we will describe how the global shock reflection-diffraction problems can be formulated as free boundary problems for nonlinear conservation laws of mixed-composite hyperbolic-elliptic type. Finally we will discuss some recent developments in attacking the shock reflection-diffraction problems, including the existence, stability, and regularity of global regular configurations of shock reflection-diffraction by wedges. The approach includes techniques to handle free boundary problems, degenerate elliptic equations, and corner singularities, which is highly motivated by experimental, computational, and asymptotic results. Further trends and open problems in this direction will be also addressed. This talk will be mainly based on joint work with M. Feldman.