Study on Mach Reflection and Mach Configuration

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Shock reflection is a fundamental phenomenon in studying compressible flow. Since Ernst Mach first studied the reflection phenomena of shock waves in 1878 [6] great amount of research works on such phenomena, mainly experiments and computations, was proceeded. In gas dynamics when a shock wave meets an obstacle in its propagation process the shock reflection occurs generally. There are various different shock reflection patterns, depending on the shape of the obstacle, the angle of the incident shock front hitting the obstacle, as well as the parameters of the incident shock.

A typical problem in studying the shock reflection is that a plane shock with a constant speed runs into a ramp. In 1943 von Neumann [8] mathematically modeled the regular reflection (RR) and Mach reflection (MR) for the problem on moving plane shock attacking an inclined ramp. He found that when the angle of the shock front with the ramp is smaller than a critical value, then the regular reflection occurs. In this case, after the instant of incident shock's hitting the ramp, the reflected shock forms an expanding bubble. On the other hand, if the angle of the shock front with the ramp is greater than the critical value, the Mach reflection occurs. In this case the incident shock and the reflected shock will only meet at a point away from the ramp, and there appears another shock front called Mach stem connecting the intersection and the ramp. The appearance of Mach stem forms a triple shock reflection. Besides, according to the analysis of shock theory one can confirm that there is another contact discontinuity issuing from the triple point. Such a construction of nonlinear waves is called Mach configuration, which is the core of Mach reflections.

The regular reflection and the Mach reflection can also be observed in steady compressible flow. Consider a plane shock hitting a wall in stationary flow. When the angle between the incident shock and the wall is smaller than a critical value determined by the parameters of flow, then a regular reflection occur. The reflected shock is an oblique shock, which is also a plane shock starting from the intersection of the incident shock and the reflected shock. However, when this angle is greater than the critical value then a Mach reflection occurs. Like the situation for the unsteady flow one can also find the triple point associating three shock waves and the Mach configuration near the point.

The flow ahead of the incident shock is supersonic. Since the Mach stem is approximately perpendicular to the velocity of the flow, then the flow is always subsonic behind the Mach stem. However, behind the reflected shock, the flow can be either subsonic or supersonic. For the unsteady flow, if the flow behind the reflected shock is subsonic, then the reflection pattern is called Simple Mach reflection (SMR). Moreover, if the flow behind the reflection shock is supersonic, then the shock reflection pattern will be more complicated. A kink on the reflected shock appears. In this situation the Mach reflection is called Transition Mach Reflection (TMR) or Double Mach Reflection (DMR), depending on the relative velocity of the flow with respect to the kink [1],[7]. Particularly, in DMR case one Mach reflection will contain two Mach configurations (that is the reason why one call the reflection pattern by *double* Mach reflection). Correspondingly, in stationary flow case, the Mach reflection can also be divided to two types. One is (EE) type: behind the reflected shock the flow is subsonic; and the other is (EH) type: behind the reflected shock the flow is supersonic.

The most analysis on the Mach reflection is based on the von Neumann's model, which can be described by using shock polars. On (p, θ) coordinates system let (p_0, θ_0) stands for the state ahead of the shock front, then all possible state (p, θ) behind the shock front forms a locus by using the Rankine-Hugoniot conditions with (p_0, θ_0) as its self-intersection. Denote the shock polar corresponding to the incident shock front by I-polar. For any point (p_1, θ_1) on I-polar, if the corresponding velocity is supersonic, then we can draw another shock polar with self-intersection (p_1, θ_1), which is called R-polar. The shock reflection pattern seriously depends on the relative position of these two curves. For instance, if the R-polar intersects the p-axis, then the regular reflection is possible. On the other hand, if the I-polar and the R-polar has an intersection other than (p_1, θ_1), then Mach reflection is possible theoretically.

However, the possibility of some pattern of shock reflection does not mean that such a reflection will actually occur. Though in many cases the appearance of RR and MR can be predicted by the shock theory and the theoretical prediction well agree with experiments. There are also some cases (particularly, when the incident shock is weak) the prediction disagrees with experiments. RR and MR may apparently persist into regions where von Neumann's model has no realistic prediction. This fact, which was named as "von Neumann paradox" by Birkhoff in 1950, puzzled many researchers.

In 1990 Collera and Henderson gave a new explanation to such phenomena in [5] based on their sophisticated numerical computations. Their viewpoint is that in the above-mentioned puzzled case the wave configuration is not a simple Mach Reflection. Instead, the reflected shock wave degenerates to a compressible wave , and the so called "triple point" in this case is not a well defined single point, but an intersection between a compression wave and the incident shock wave. The compression wave forms a shock wave afterwards, which likes a part of the reflected shock in Mach reflection case. This wave configuration is called by von Neumann reflection. However, due to the limitation of the precision of experiments the recent experiments could not verify such a wave structure.

A serious mathematical analysis on Mach reflection is given in [2] for stationary compressible flow and in [3] for pseudo-stationary compressible flow. We studied the case when the flow is subsonic (or relatively subsonic) behind the reflected flow, and proved the stability of Mach Configuration in this case. For stationary case it is proved that for a given flat Mach configuration of EE type, if the upstream flow ahead of the incident shock and the incident shock itself are perturbed, then the structure of the whole configuration holds, while the other two shock fronts and the slip line, as well as the flow field in all regions are also perturbed. For the pseudo-stationary compressible flow we have the similar conclusion, which is available to describe Simple Mach Reflection. Therefore, our conclusion on stability confirms the reasonableness of Mach configuration of (EE) type in stationary case and the local structure of (SMR) in pseudo-stationary case.

The main approach in these works is as follows. The free boundary value problem is divided to two subproblems: the first one is to solve a fixed boundary value problem of Euler system with a fixed approximate shock front as a part of the boundaries of the domain, and the second one is to update the location of shock front. These two steps will reduce the process of solving the free boundary value problem to looking for a fixed point of a map updating the position of shock front.

Another difficulty is that the slip line is also to be determined. This line is a characteristics carrying discontinuity of density and velocity. To fix this unknown contact discontinuity we introduce new coordinates like Lagrange coordinates in one dimensional unsteady flow. For pseudo-stationary flow we have also to introduce an additional unknown function to realize the Lagrange coordinates. Under such a coordinates transformation all stream line are straightened, so that the unknown slip line coincides with the coordinate axis.

The stationary Euler system in subsonic region is a nonlinear elliptic-hyperbolic composed system. To deal with such a composed system we reduce the system to a canonical form, which decouples the elliptic part and the hyperbolic part of the system in the principal part. Then these two parts can be treated by using elliptic technique and hyperbolic technique separately, so that the estimates and existence of solution to the whole composed system can be established by combining the analysis for these two parts. Moreover, because of the appearance of corners of the domain more effort should be paid to obtain estimates for the elliptic problem.

Finally, let us list some open problems in the end of this abstract.

1. The stability of Mach configuration of (EH) type. The problem likes a Tricomi problem for a Lavrentiev-Bitsadze equation in a domain with free boundary.

2. The global existence of Mach reflection. In pseudo-stationary case the global existence involves the development of slip line, which is a vortex sheet indeed. In stationary case, a part of this problem is determining the location of the triple point or the height of the Mach stem.

3. The transition between RR and MR. Shock polar analysis gives some region of the parameters of the flow, where both RR and MR are possible. Which one occurs in practice? Does the occurrence rely on both flow parameters and the process of the change of parameters? Collera and Henderson gave an explanation on von Neumann paradox, could the von Neumann configuration be verified by mathematical analysis?

4. The study on the more complicated pattern of Mach reflections, like (TMR) or (DMR). It is obviously a part of the problems on global existence of Mach reflection.

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